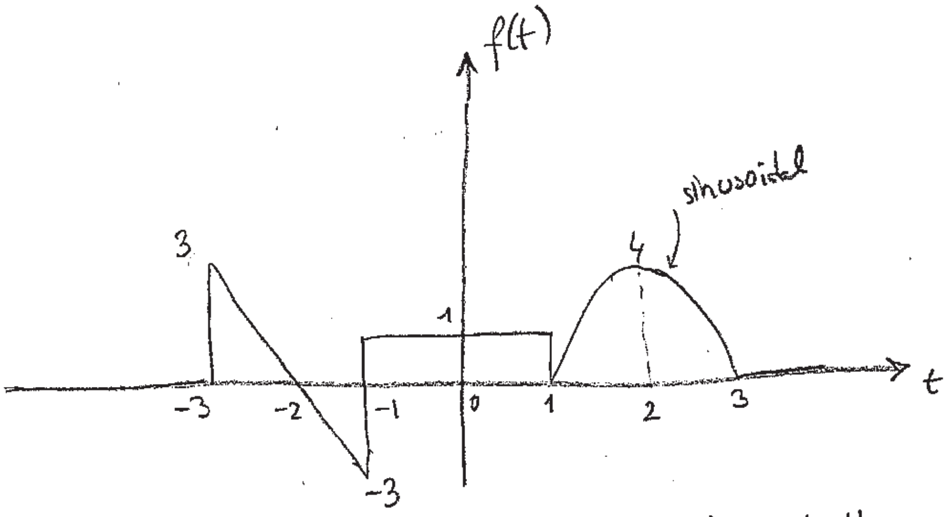
EE-201 Topics to be covered. (Sencer Koç)
Fall 2013-14

Example:



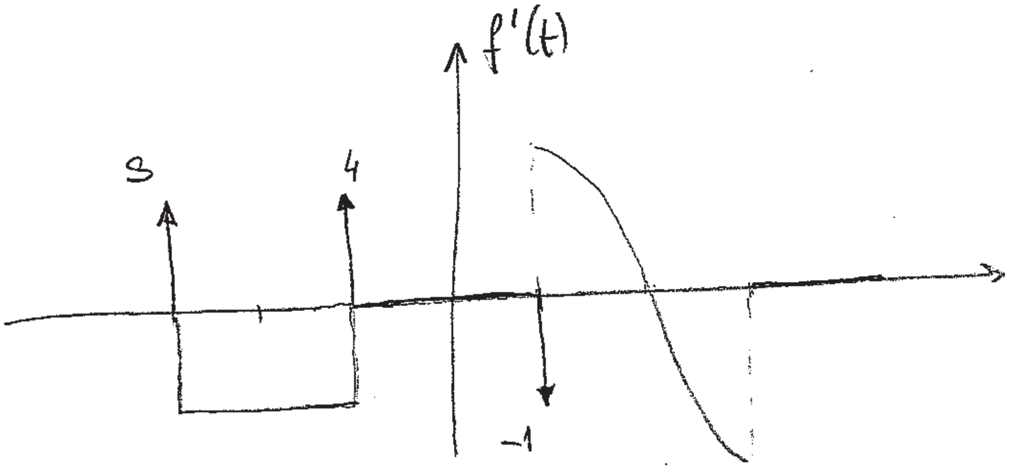
Write $f(t)$ and determine $f'(t)$, both graphically & analytically.

Sltn: Define $p(t;a,b) = u(t-a) - u(t-b)$, which is 1 for $a<t<b$ and zero elsewhere.

$$f(t) = (-3t-6)\,p(t;-3,-1) + p(t;-1,1) + 4\sin\left(\frac{\pi}{2}(t-1)\right)p(t;1,3)$$

$$= (-3t-6)\left[u(t+3)-u(t-1)\right] + \left[u(t+1)-u(t-1)\right]$$

$$+ 4\sin\left(\frac{\pi}{2}(t-1)\right)\left[u(t-1)-u(t-3)\right]$$



Analytically, we have

$$f'(t) = (-3)\,p(t;-3,-1) + (-3t-6)\,p'(t;-3,-1) + p'(t;-1,1)$$
$$+ \frac{4\pi}{2}\cos\left(\frac{\pi}{2}(t-1)\right) p(t;1,3) + 4\sin\left(\frac{\pi}{2}(t-1)\right) p'(t;1,3)$$

Note that

$$p'(t;a,b) = \left[u(t-a) - u(t-b)\right]' = \delta(t-a) - \delta(t-b)$$

Then:

$$f'(t) = -3\left[u(t+3) - u(t-1)\right] + \overbrace{(-3t-6)\,\delta(t+3)}^{3\delta(t+3)} - \overbrace{(-3t-6)\,\delta(t+1)}^{3\delta(t+3)}$$
$$+ \delta(t+1) - \delta(t-1) + \frac{4\pi}{2}\cos\left(\frac{\pi}{2}(t-1)\right)\left[u(t-1) - u(t-3)\right]$$
$$+ \underbrace{4\sin\left(\frac{\pi}{2}(t-1)\right)\delta(t-1)}_{0} + \underbrace{4\sin\left(\frac{\pi}{2}(t-1)\right)\delta(t-3)}_{0}$$

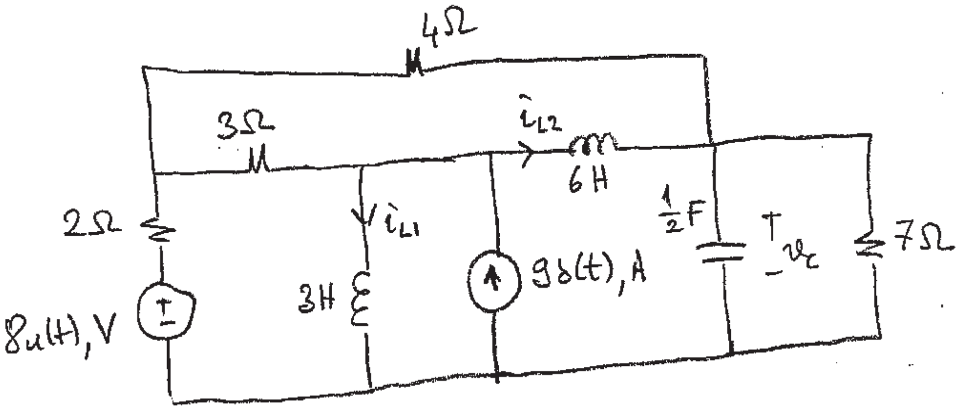
Remembering that

$$f(t)\,\delta(t-a) = f(a)\,\delta(t-a)$$

we can write:

$$f'(t) = -3\left[u(t+3) - u(t-1)\right] + 2\pi\cos\left(\frac{\pi}{2}(t-1)\right)\left[u(t-1) - u(t-3)\right]$$
$$+ 3\delta(t+3) + 4\delta(t+1) - \delta(t-1)$$

Example: (2PS V, 6c)

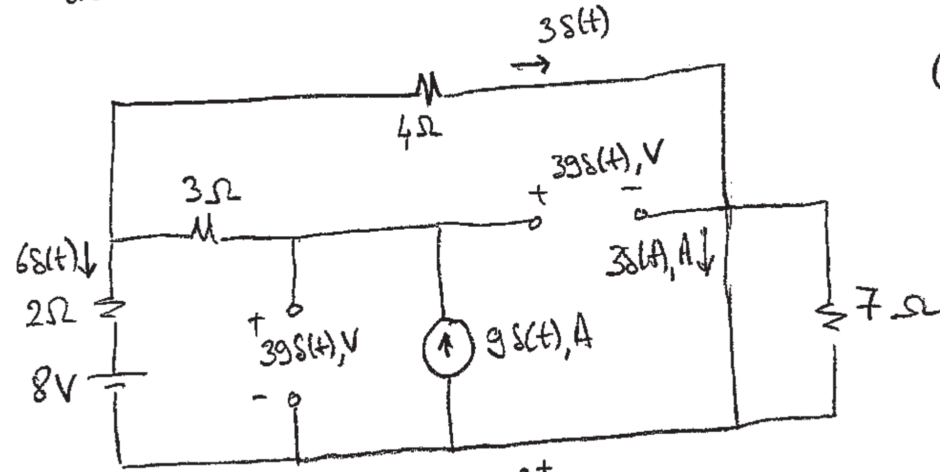


$v_c(0^-)=6V$, $i_{L1}(0^-)=3A$, $i_{L2}(0^-)=6A$

Find $v_c(0^+)$, $i_{L1}(0^+)$, $i_{L2}(0^+)$, $v_c(\infty)$, $i_{L1}(\infty)$, $i_{L2}(\infty)$.

Sltn: Assume all initial values are zero. The step voltage input does not cause a jump in state variables. However, for the impulse we can draw the following equivalent at $t=0^+$

@ $t=0^+$

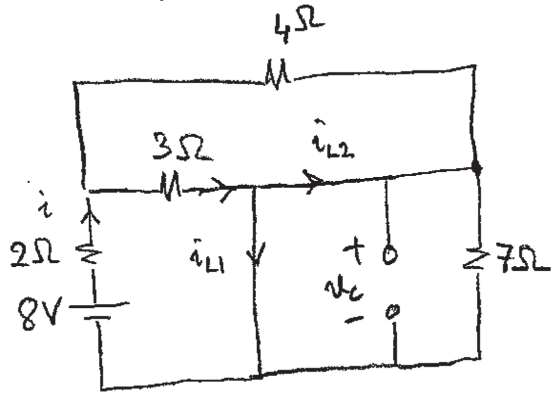


$$v_c(0^+) = v_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} 3\delta(t)\,dt = 6 + 2\cdot 3 = 12V$$

$$i_{L1}(0^+) = i_{L1}(0^-) + \frac{1}{L_1}\int_{0^-}^{0^+} 39\,\delta(t)\,dt = 3 + \frac{39}{3} = 16A$$

$$i_{L2}(0^+) = i_{L2}(0^-) + \frac{1}{L_2}\int_{0^-}^{0^+} 39\,\delta(t)\,dt = 6 + \frac{39}{6} = 12.5\ A$$

The equivalent circuit at $t \to \infty$ is:



$v_c(\infty) = 0$

$i_{L1}(\infty) = \frac{56}{26} A = 2.154 A$

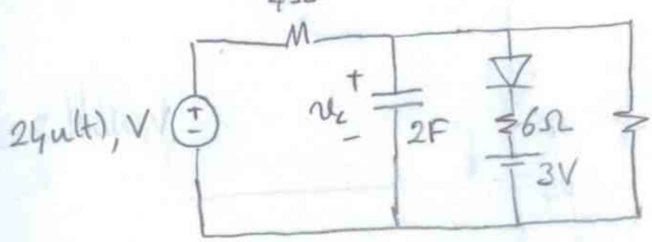
$i_{L2}(\infty) = -\frac{24}{26} A = -0.923 A$

$R_{eq} = 2 + 3 \| 4 = \frac{26}{7} \Omega$

$i = \frac{8}{R_{eq}} = \frac{56}{26} A = i_{L1}$

$i_{L2} = -i_{4\Omega} = -\frac{3}{7} i = -\frac{24}{26} A$

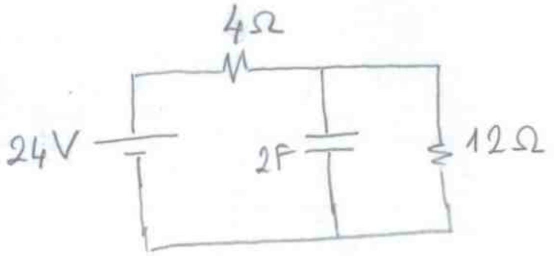
Example:



$v_c(0^-) = 0$

Find $v_c(t)$ for $t \geq 0$.

$v_c(0^+) = 0 \Rightarrow$ D is OFF



$R_1 = 4 // 12 = 3\Omega$

$\tau_1 = 6$ sec.

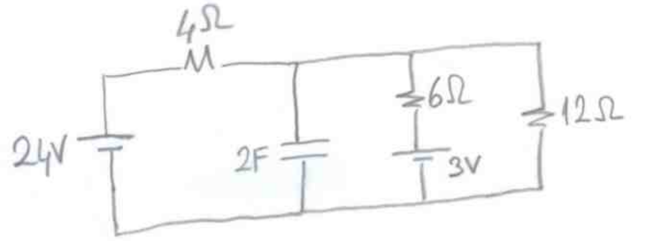
$v_c(\infty) = \frac{12}{16} \cdot 24 = 18\,V$

$v_c(t) = 18 - 18\,e^{-t/\tau_1}$, V.

D will turn on when $v_c(t_1) = 3$.

$18 - 18\,e^{-t_1/\tau_1} = 3 \Rightarrow t_1 = \tau_1 \ln\frac{5}{6} \cong 0.182\,\tau_1 = 1.09$ sec.

For $t > t_1$, the circuit becomes:



$R_2 = 2\Omega$

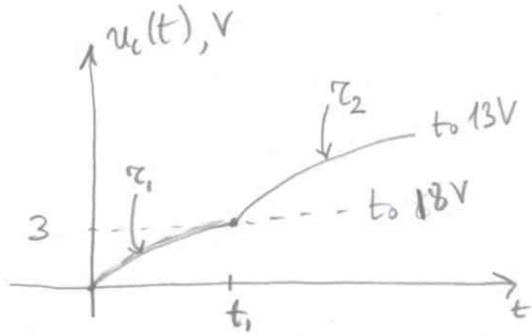
$\tau_2 = 4$ sec

$v_c(\infty) = \frac{4}{8} 24 + \frac{3}{9} \cdot 3 = 13V$.

$v_c(t_1) = 3V$

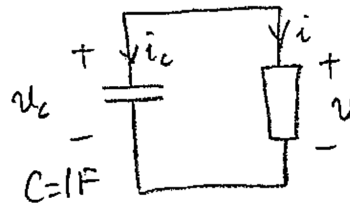
$v_c(t) = 13 - 10\,e^{-(t-t_1)/\tau_2}$, V

$$v_c(t) = \begin{cases} 6 - 6\,e^{-t/\tau_1}, & 0 \leq t \leq t_1 \\ 13 - 10\,e^{-(t-t_1)/\tau_2}, & t \geq t_1 \end{cases}$$

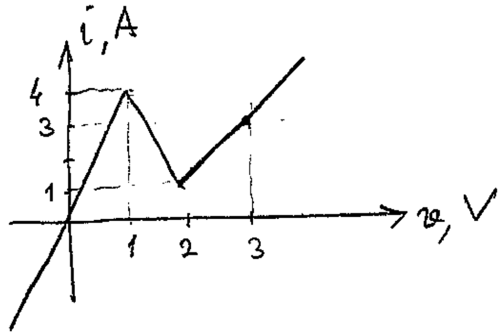


$\tau_1 = 6$ sec; $\tau_2 = 4$ sec; $t_1 = \tau_1 \ln\frac{5}{6}$

Example:



Non-linear element



$v_c(0) = 4V.$

Find $v_c(t)$.

Sltn: At $t=0$ $V = 4V \Rightarrow i = 5A$

i) For $2 \le v \le 4$ the device characteristics is $i = 2\left(v - \frac{3}{2}\right)$

$$C\frac{dv_c}{dt} = -i \Rightarrow \frac{dv_c}{dt} + 2v_c = 3$$

$$v_c(0) = 4$$

$$v_c(t) = \left(4 - \frac{3}{2}\right)e^{-2t} + \frac{3}{2} = \frac{5}{2}e^{-2t} + \frac{3}{2}, V \qquad 0 < t < t_1$$

$$v_c(t_1) = 2 \Rightarrow t_1 = \frac{-1}{2}\ln\frac{1}{5} \cong 0.8 \text{ sec}$$

ii) For $1 \le v \le 2$ the device characteristics is $i = -3\left(v - \frac{7}{3}\right)$

$$C\frac{dv_c}{dt} = -i \Rightarrow \frac{dv_c}{dt} - 3v_c = -7$$

$$v_c(t_1) = 2v$$

$$v_c(t) = \left(2 - \frac{7}{3}\right)e^{3(t-t_1)} + \frac{7}{3} = -\frac{1}{3}e^{3(t-t_1)} + \frac{7}{3}, V \qquad t_1 \le t \le t_2$$

$$v_c(t_2) = 1 \Rightarrow t_2 = t_1 + \frac{1}{3}\ln 4 \cong 1.27 \text{ sec.}$$

iii) For $v < 1$ the device characteristics is $i = 4v$

$$\left.\begin{aligned} \frac{dv_c}{dt} &= -4v_c \\ v_c(t_2) &= 1 \end{aligned}\right\} \Rightarrow v_c(t) = e^{-4(t-t_2)}, V \quad \text{for} \quad t \geq t_2$$

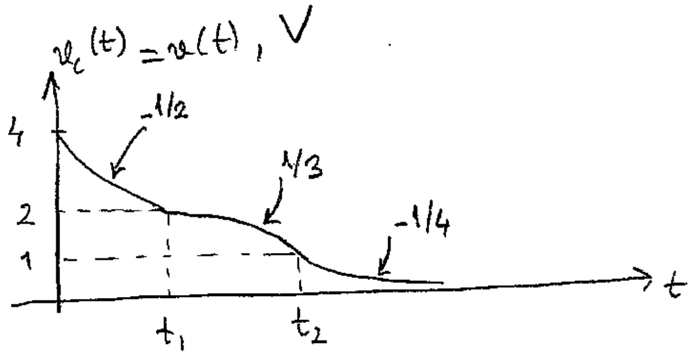
$v(t) \to 0$ as $t \to \infty$

We can write:

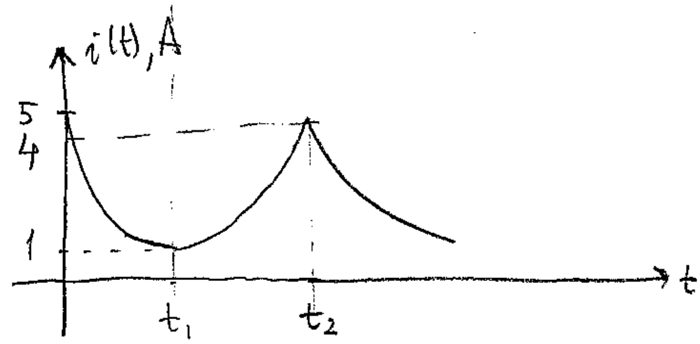
$$v_c(t) = \begin{cases} \frac{5}{2}e^{-2t} + \frac{3}{2}, & 0 \leq t \leq t_1 \\ -\frac{1}{3}e^{3(t-t_1)} + \frac{7}{3}, & t_1 \leq t \leq t_2 \\ e^{-4(t-t_2)}, & t \geq t_2 \end{cases}$$

or

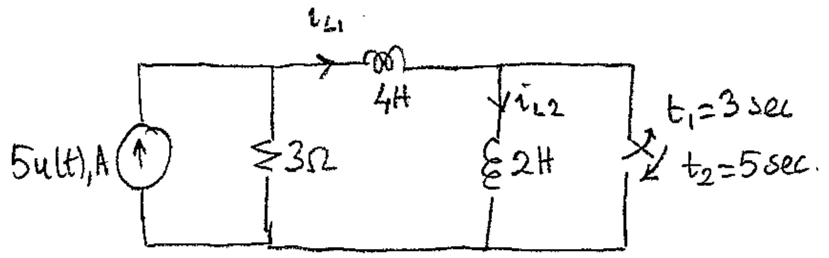
$$v_c(t) = \left(\frac{5}{2}e^{-2t} + \frac{3}{2}\right)\left[u(t) - u(t-t_1)\right]$$
$$+ \left(-\frac{1}{3}e^{3(t-t_1)} + \frac{7}{3}\right)\left[u(t-t_1) - u(t-t_2)\right]$$
$$+ e^{-4(t-t_2)} \, u(t-t_2)$$



$$i(t) = -C\frac{dv_c}{dt} = -\frac{dv_c}{dt}$$



Example :



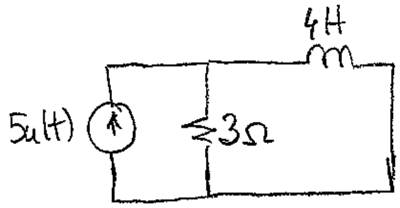
$i_{L1}(0^-) = 2A$

$i_{L2}(0^-) = 1A$

Find $i_{L1}(t)$ for $t \geq 0$

Switch is closed for $0 < t < 3$
open for $3 < t < 5$
closed for $t > 5$.

Sltn: $i_{L1}(0^+) = 2A$; $i_{L2}(0^+)$ is irrelevant since switch is closed.



$$\tau_1 = \frac{L_1}{R} = \frac{4}{3} \text{ sec.} \qquad i_{L1}(\infty) = 5$$

$$i_{L1}(t) = -3e^{-t/\tau_1} + 5 \, , \; A.$$

While the switch is closed, the current through L_2 will not change.

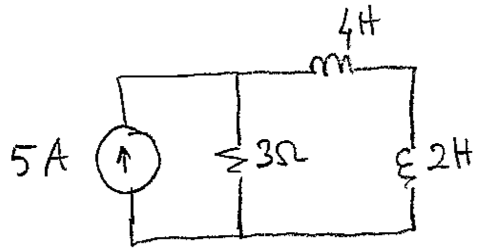
$$L\frac{di_L}{dt} = v_L = 0 \Rightarrow i_L \text{ is constant} \Rightarrow i_{L2}(3^-) = 1A$$

When the switch is opened at t_1, we have the pathalogical case and we will use conservation of flux.

$$\frac{\phi}{L_T} = \frac{\phi_1 + \phi_2}{L_T} = \frac{i_{L1}(t_1^-)L_1 + i_{L2}(t_1^-)L_2}{L_1 + L_2} = i_{L1}(t_1^+) = i_{L2}(t_1^+)$$

$$= \frac{\left(-3e^{-t_1/\tau_1} + 5\right)4 + 1 \cdot 2}{6} = 3.45A$$

For $3<t<5$, switch is open and we have:



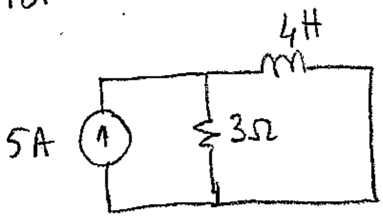
$$\tau_2 = \frac{L_T}{R} = \frac{6}{3} = 2 \text{ sec.}$$

$$i_{L1}(\infty) = 5A \; ; \quad i_{L1}(t_1^+) = 3.45A$$

$$i_{L1}(t) = -1.55\, e^{-(t-t_1)/2} + 5, \text{ A} \qquad 3<t<5$$

$$i_{L1}(t_2) = -1.55\, e^{-2/2} + 5 = 4.43\,A.$$

For $t>5$

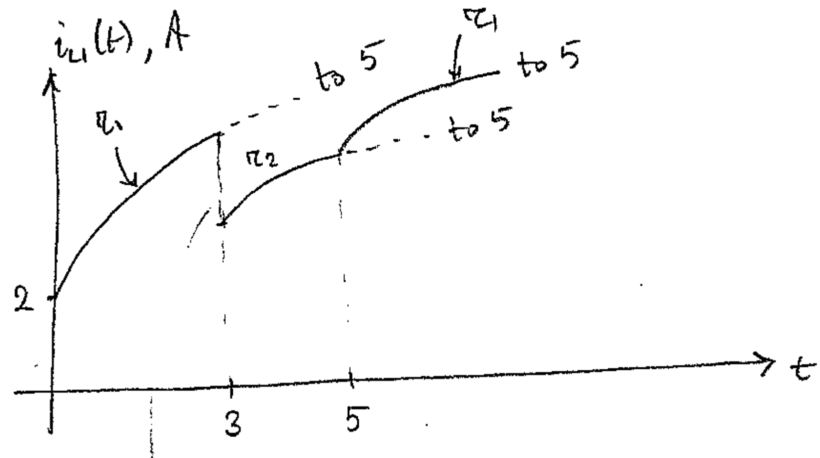


$$\tau_1 = \frac{L_1}{3} = \frac{4}{3} \text{ sec}$$

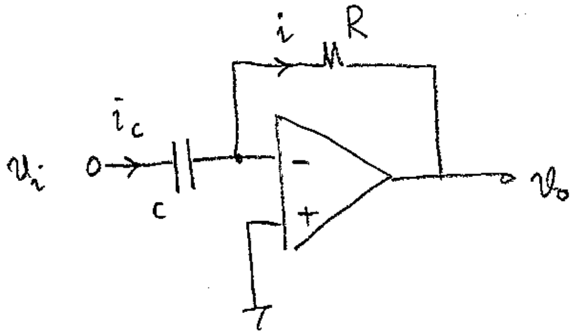
$$i_{L1}(\infty) = 5A \, , \quad i_{L1}(t_2^+) = 4.43\,A.$$

$$i_{L1}(t) = -0.57\, e^{-(t-t_2)/\tau_1} + 5, \text{ A} \qquad t>5.$$

$$i_{L1}(t) = \begin{cases} -3e^{-t/\tau_1} + 5, & 0<t<t_1 \\ -1.55\, e^{-(t-t_1)/\tau_2} + 5, & t_1<t<t_2 \\ -0.57\, e^{-(t-t_2)/\tau_1} + 5, & t>t_2 \end{cases}$$



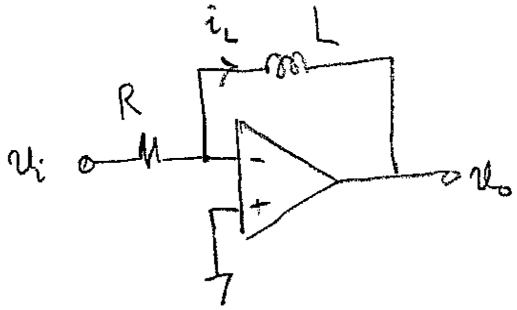
pathological case where energy is not conserved!



$$i_c = C\frac{dv_i}{dt} = i$$

$$v_o = -iR = -RC\frac{dv_i}{dt}.$$

Therefore, provided that the OP-AMP does not saturate, the output is the derivative of the input. This circuit is called a differentiator.

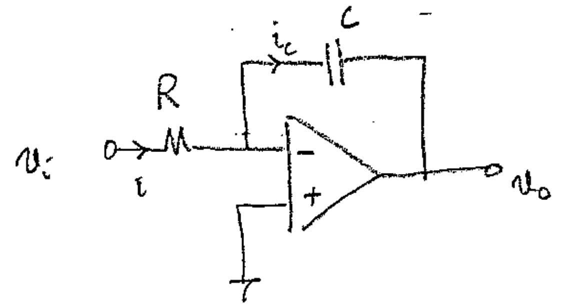


$$v_o = -v_L = -L\frac{di_L}{dt}$$

$$i_L = \frac{v_i}{R}$$

$$\therefore \quad v_o = -\frac{L}{R}\frac{dv_i}{dt}$$

Again we have a differentiator, provided that the OP-AMP does not saturate.



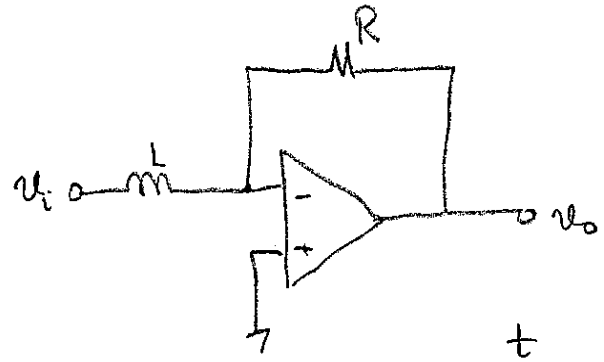
$$i = i_c = \frac{v_i}{R}$$

$$i_c = C\frac{dv_c}{dt}$$

$$v_o = -v_c$$

$$v_c(t) = v_c(0) + \frac{1}{C}\int_0^t i_c(\tau)\,d\tau = v_c(0) + \frac{1}{RC}\int_0^t v_i(\tau)\,d\tau = -v_o(t)$$

Hence, we have an integrator, provided that the OP-AMP does not saturate.



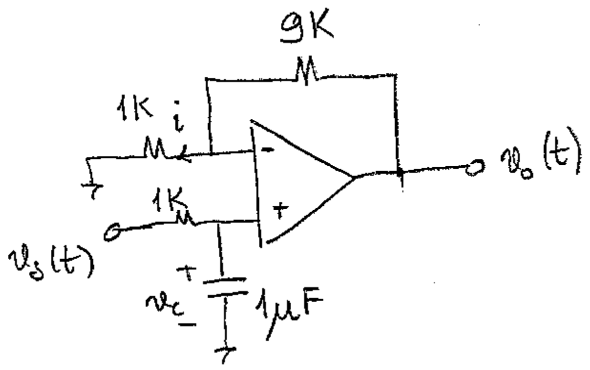
$$v_L = v_i = L\frac{di_L}{dt}$$

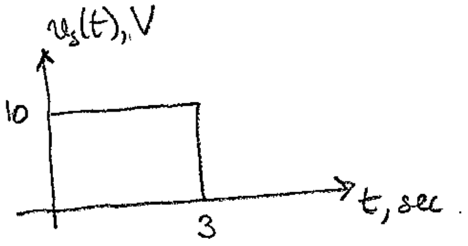
$$v_o = -R\,i_L$$

$$v_o(t) = -R\,i_L(0) - \frac{R}{L}\int_0^t v_i(\tau)\,d\tau.$$

Again, an integrator.

Example:





$v_c(0^-)=0$.

$E_s = 20$ V

Determine $v_o(t)$.

$$v_+ = v_c = 10 - 10\,e^{-t/\tau}\ ,\ V \qquad \tau = 1K \times 1\mu F = 1\,msec.$$

$$i = \frac{v_c}{1K} = 10\,(1 - e^{-t/\tau})\ ,\ mA$$

$$v_o = v_c + 9i = 100\,(1 - e^{-t/\tau})\ ,\ V$$

$$100\,(1 - e^{-t_1/\tau}) = 20 \Rightarrow t_1 = -\tau \ln\frac{4}{5} \cong 0.223\,\tau = 223\,\mu sec.$$

$$v_c(3) = 10\,(1 - e^{-3000}) = 10V.$$

For $t>3$, capacitor discharges through 1K resistor.

$$v_c(t) = 10\,e^{-(t-3)/\tau}\ ,\quad t>3.$$

$v_o(t) = 10\,v_c(t)$, hence will be sat initially.

$$100\,v_c(t_2) = 20 \Rightarrow t_2 = 3 + \tau \ln 5 = 3 + 1.61\ msec.$$